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# ELASTICITY MODULI AND POLYTROPIC INDICES IN PROCESSES OF CHANGE OF STATES OF REAL GASES AND LIQUIDS

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*It is found that in polytropic processes of change of states of real gases and liquids the polytropic index is the ratio of the polytropic elasticity modulus and the hydrodynamic pressure. Formulas for determination of the polytropic elasticity modulus at constant polytropic efficiency as a function of the adiabatic index and the Grüneisen parameter are obtained.*

It is known that in gases and liquids the elasticity modulus $K = -vdp/dv$ is determined by the value of the derivative $dp/dv$, depending on the conditions under which the state of the gas or liquid changes [1]. The isothermal elasticity modulus $K_T$ and the adiabatic elasticity modulus $K_s$ are used most frequently. The elasticity moduli in basic thermodynamic processes of a real gas and liquid depend on the values of the adiabatic index and the Grüneisen parameter $\Gamma$ [2-5]:

$$\Gamma = \alpha a^2/c_p = (c_p/c_v - 1)/(\alpha T) = (\rho/T)\,(\partial T/\partial \rho)_s\,. \tag{1}$$

In basic thermodynamic processes of real gases and liquids

$$K_T = K_s c_v/c_p = kp/[\Gamma\alpha T + 1]\,;\quad K_s = \rho a^2 = kp\,;$$

$$K_h = kp/(\Gamma + 1)\,;\quad K_u = (k - \Gamma)p\,.$$

In an ideal gas, $\Gamma = k - 1$; $K_T = K_h = K_u = p$; $K_s = (c_p/c_v)p$.

In polytropic processes, the values of the derivative $dp/dv$ depend on the polytropic efficiency in the compression process $\eta_{\text{c.p}} = vdp/dh$ or in the expansion process $\eta_{\text{exp.p}} = dh/(vdp)$. In a compression process with a constant polytropic efficiency $\eta_{\text{c.p}}$, the polytropic index is $n_{\text{c}} = -(v/p)(\partial p/\partial v)_\eta = K_{\text{c.p}}/p$, i.e., the polytropic elasticity modulus in compression is $K_{\text{c.p}} = n_{\text{c}}p$. In an expansion process with a constant polytropic efficiency $\eta_{\text{exp.p}}$, the polytropic index is $n_{\text{exp}} = -(v/p)(\partial p/dv)_\eta = K_{\text{exp.p}}/p$, i.e., the polytropic elasticity modulus in expansion is $K_{\text{exp.p}} = n_{\text{exp}}p$.

In equilibrium processes of real gases and liquids, the following thermodynamic relations hold [6-8]:

$$\begin{gathered} dh = vdp/\eta_{\text{c.p}} = c_p dT/(1 + x\eta_{\text{c.p}}) = c_p dT/[1 + \eta_{\text{c.p}}(\alpha T - 1)]\,; \\ dT/T + dz/z = [(n_{\text{c}} - 1)/n_{\text{c}}]\,dp/p\,; \\ (n_{\text{c}} - 1)/n_{\text{c}} = [(x + 1)\,pdT]/(Tdp) + 1 - y = \alpha TpdT/(Tdp) + (m - 1)/m\,; \\ n_{\text{c}} = (1 + x)/\{y\,[(c_v/c_p)\,(1/\eta_{\text{c.p}} + x) - (1/\eta_{\text{c.p}} - 1)]\}\,; \end{gathered} \tag{2}$$

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TABLE 1. Elasticity Moduli and Polytropic Indices at $\eta_{c.p} = \eta_{exp.p} = 0.85$ for Some Gases and Liquids

| Substance | $T$, K | $p$, MPa | k, $c_p/c_v$ | $\alpha T$, $\Gamma$ | $K_s$, MPa, $K_T$, MPa | $K_h$, MPa $K_u$, MPa | $n_c$, $K_{c.p}$, MPa | $n_{exp}$, $K_{exp.p}$, MPa |
|---|---|---|---|---|---|---|---|---|
| Air | 290 | 0.1 | 1.4 | 1.003 | 0.14 | 0.1 | 1.506 | 1.321 |
| | | | 1.4 | 0.4 | 0.1 | 0.1 | 0.151 | 0.132 |
| Products of combustion of kerosene | 1000 | 0.5 | 1.3 | 1.00 | 0.65 | 0.5 | 1.373 | 1.244 |
| | | | 1.3 | 0.3 | 0.5 | 0.5 | 0.686 | 0.622 |
| Water | 293 | 0.1 | 22,110 | 0.061 | 2211 | 1994 | 22,540 | 21,750 |
| | | | 1.007 | 0.109 | 2196 | 2211 | 2254 | 2175 |
| | 293 | 50 | 49.8 | 0.082 | 2490 | 2135 | 51.32 | 48.58 |
| | | | 1.014 | 0.168 | 2456 | 2482 | 2566 | 2429 |
| Fuel RT | 293 | 0.1 | 10,760 | 0.27 | 1076 | 655 | 12,130 | 9818 |
| | | | 1.174 | 0.64 | 916 | 1076 | 1213 | 981.8 |
| Hydrogen | 20 | 0.5 | 192.6 | 0.31 | 96.3 | 30.0 | 315.7 | 144.6 |
| | | | 1.68 | 2.21 | 57.5 | 95.2 | 157.9 | 72.3 |
| Oxygen | 95 | 1.0 | 754.6 | 0.43 | 754.6 | 267.6 | 1117 | 591.4 |
| | | | 1.79 | 1.82 | 422 | 752.8 | 1117 | 591.4 |
| Propane | 300 | 10 | 36.84 | 0.743 | 368.4 | 217.5 | 41.98 | 33.37 |
| | | | 1.516 | 0.694 | 243 | 361.5 | 419.8 | 333.7 |

$$c_p - c_v = d\,(pv)/dT = Rz\,(1 + x)^2/y = mRz\,(\alpha T)^2\,;$$

$$pv/(c_pT) = Rz/c_p = [(c_p/c_v) - 1]/[\mathrm{k}\,(\alpha T)^2] = \Gamma/(\mathrm{k}\alpha T)\,.$$

As a result of solution of the system of equations (2), equalities are established for the process of polytropic compression:

$$\eta_{c.p} = \Gamma n_c/[n_c\,(\Gamma + 1) - \mathrm{k}]\,;\quad (n_c - 1)/n_c = [\Gamma + \eta_{c.p}\,(\mathrm{k} - \tilde{A} - 1)]/(\mathrm{k}\eta_{c.p})\,;$$

$$n_c = \mathrm{k}\eta_{c.p}/[\eta_{c.p} - \Gamma\,(1 - \eta_{c.p})]\,. \tag{3}$$

Similarly, for the process of polytropic expansion we obtain

$$\eta_{exp.p} = 1 - (\mathrm{k} - n_{exp})/(n_{exp}\,\Gamma)\,;\quad (n_{exp} - 1)/n_{exp} = [\mathrm{k} - 1 - \Gamma\,(1 - \eta_{exp.p})]/\mathrm{k}\,;$$

$$n_{exp} = \mathrm{k}/[1 + \Gamma\,(1 - \eta_{exp.p})]\,. \tag{4}$$

In an ideal gas, formulas (3) and (4) acquire the form

$$\eta_{c.p} = n_c\,(\mathrm{k} - 1)/[\mathrm{k}\,(n_c - 1)]\,;\quad (n_c - 1)/n_c = (\mathrm{k} - 1)/(\mathrm{k}\eta_{c.p})\,;$$

$$n_c = \mathrm{k}\eta_{c.p}/[1 - \mathrm{k}\,(1 - \eta_{c.p})]\,;$$

$$\eta_{exp.p} = k\,(n_p - 1)/[n_p\,(k - 1)]\,;\quad (n_p - 1)/n_p = (k - 1)\,\eta_{exp.p}/k\,; \tag{5}$$

$$n_p = k/[k - \eta_{exp.p}\,(k - 1)]\,.$$

Table 1 presents the parameters k, $c_p/c_v$, $\alpha T$, $\Gamma$, the elasticity moduli $K_s$, $K_T$, $K_h$, $K_u$, $K_{c.p}$, $K_{exp.p}$, and the polytropic indices $n_c$ and $n_{exp}$ for air [9], the products of combustion of kerosene with a coefficient of excess air $\alpha$ = 1.1 [10], water [11], aviation fuel RT [12], and liquid hydrogen [13], oxygen [14], and propane [15] at $\eta_{c.p} = \eta_{exp.p} = 0.85$.

Use of expressions (5) for determination of the efficiencies $\eta_{c.p}$ and $\eta_{exp.p}$ in analysis of the processes of compression and expansion of a real gas can lead to large errors. For example, in polytropic compression of air with $T_1$ = 280 K, $p_1$ = 4.838 MPa, $z_1$ = 0.982, and $\rho_1$ = 61.3 kg/m$^3$ to $p_2$ = 50 MPa and $T_2$ = 750 K, where $z_2$ = 1.21 and $\rho_2$ = 193.2 kg/m$^3$, the index $n_c$ is equal to 2.035. According to data of [9], $k_m$ = 1.6 and $\Gamma_m$ = 0.45. We find $\eta_{c.p}$ = 0.678 by formula (3). From relation (5), $\eta_{c.p}$ = 0.737 for an ideal gas. The error of determining $\eta_{c.p}$ by (5) amounts to 8.7%. At a constant efficiency $\eta_{c.p}$ = 0.678 and $k_m$ = 1.6, from expression (5) the index $n_c$ = 2.238, the density $\rho_2$ = 174.1 kg/m$^3$. The error of determining $\rho_2$ by (5) is 9.9%.

In polytropic expansion of air with $p_1$ = 26 MPa, $T_1$ = 550 K, $z_1$ = 1.12, and $\rho_1$ = 147 kg/m$^3$ to $p_2$ = 2.5 MPa and $T_2$ = 350 K, where $z_2$ = 1.002 and $\rho_2$ = 24.84 kg/m$^3$, the index $n_{exp}$ is 1.317. According to the data of [9], $k_m$ = 1.54 and $\Gamma_m$ = 0.435. We find $\eta_{exp.p}$ = 0.611 by formula (5). From relation (5), for an ideal gas $\eta_{exp.p}$ = 0.686. The error of determining $\eta_{exp.p}$ by (5) is 12.3%. At a constant efficiency $\eta_{exp.p}$ = 0.611 and $k_m$ = 1.54, from expression (5) the index $n_{exp}$ = 1.274, and the density $\rho_2$ = 23.24 kg/m$^3$. The error of determining $\rho_2$ by (5) is 6.4%.

As is known, the polytropic engineering work of a compressor

$$L = -\int_1^2 v dp = [np_1\,v_1/(n - 1)]\,[(p_2/p_1)^{(n-1)/n} - 1]$$

differs from the work spent on compressing the gas in the compressor

$$L_{1-2} = \int_1^2 p dv = [p_1\,v_1/(n - 1)]\,[(p_2/p_1)^{(n-1)/n} - 1]\,.$$

The same magnitude of engineering work is obtained by the formula

$$L = \int_1^2 K_{c.p}\,dv = \int_1^2 npdv\,.$$

Similarly, the polytropic engineering work of a turbine

$$L = -\int_1^2 v dp = [np_1 v_1/(n - 1)]\,[1 - (p_2/p_1)^{(n-1)/n}]$$

also differs from the work of expansion of the gas in the turbine:

$$L_{1-2} = \int_1^2 p dv = [p_1 v_1/(n - 1)]\,[1 - (p_2/p_1)^{(n-1)/n}]\,.$$

The same magnitude of engineering work is obtained by the formula

$$L = \int_1^2 K_{\text{exp.p}}\, dv = \int_1^2 npdv\,.$$

Consequently, the polytropic engineering work of a compressor or a turbine can be presented as the work of compression or expansion of a gas with a pressure equal to the polytropic elasticity modulus $K_{\text{c.p}}$ or $K_{\text{exp.p}}$.

Since the Euler number Eu = $p/(\rho w^2)$ = $1/(\text{kM}^2)$ is equal to the ratio of the force of hydrodynamic pressure to the inertial force, the adiabatic index k = $K_s/p$ can be assumed to be the ratio of the hypothetical "force of adiabatic elasticity" to the force of hydrodynamic pressure, and the square of the Mach number $\text{M}^2$ = $\rho w^2/(\rho a^2)$ is the ratio of the inertial force to the "force of adiabatic elasticity." In this interpretation, all the main criteria of hydrodynamic similarity Sh, Fr, Re, Eu, k, and $\text{M}^2$ are ratios of forces acting in flows of a gas or a liquid.

**Conclusions.** The polytropic index in processes of change of states of real gases or liquids at constant efficiency is the ratio of the polytropic modulus of elasticity and the hydrodynamic pressure. Polytropic indices of the processes of compression and expansion of real gases and liquids depend on the polytropic efficiency and the values of the adiabatic index and the Grüneisen parameter. Use of formulas (5) for determination of the polytropic indices of a process with a known polytropic efficiency or of the polytropic efficiency with a known polytropic index of the process in a real gas can lead to large errors.

## NOTATION

$v$, specific volume; $p$, pressure; $\rho$, density; $T$, temperature; $c_p$ and $c_v$, isobaric and isochoric specific heats; $K_T = -v(\partial p/\partial v)_T$, isothermal elasticity modulus; $K_s = -v(\partial p/\partial v)_s = \rho a^2 = \text{k}p$, adiabatic elasticity modulus; $K_h = -v(\partial p/\partial v)_h$, isoenthalpic elasticity modulus; $K_u = -v\ (\partial p/\partial v)_u$, isoenergetic elasticity modulus; k = $\rho a^2/p = K_s/p = (\rho/p)(\partial p/\partial \rho)_s$, adiabatic index; $a$, velocity of sound; $\Gamma$, Grüneisen parameter; $\alpha$, isobaric coefficient of volumetric expansion; $\alpha T = \alpha/\alpha_0$; $\alpha_0 = 1/T$, coefficient $\alpha$ when $p \to 0$; $\eta_{\text{c.p}}$, polytropic efficiency in the process of compression; $K_{\text{c.p}} = -v/(\partial p/\partial v)_{\eta_{\text{c.p}}} = n_c p$, polytropic elasticity modulus in compression; $n_c = K_{\text{c.p}}/p$, polytropic index of the process of compression; $\eta_{\text{exp.p}}$, polytropic efficiency in the process of expansion; $K_{\text{exp.p}} = -v(\partial p/\partial v)_{\eta_{\text{exp.p}}} = n_{\text{exp}}p$, polytropic elasticity modulus in expansion; $n_{\text{exp}} = K_{\text{exp.p}}/p$, polytropic index of the process of expansion; $z = pv/(RT)$, compressibility factor; $R$, gas constant; $x = (T/z)(\partial z/\partial T)_p = \alpha T - 1$ and $y = 1/m = 1 - (p/z)(\partial z/\partial p)_T$, compressibility functions; $w$, flow velocity; $m = (\rho/p)(\partial p/\partial \rho)_T$, index of an isotherm; M, Mach number; Eu, Euler number; Sh, Strouhal number; Fr, Froude number; Re, Reynolds number. Subscripts: 1, at the inlet; 2, at the outlet; m, mean value of the parameter; c.p, polytropic in the process of compression; exp.p, polytropic in the process of expansion; $T$, at constant temperature; $s$, at constant entropy; $v$, at constant volume; $p$, at constant pressure; $h$, at constant enthalpy; $u$, at constant internal energy; $\eta_{\text{c.p}}$, at constant polytropic efficiency of the process of compression; $\eta_{\text{exp.p}}$, at constant polytropic efficiency of the process of expansion; 0, when $p \to 0$.

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